smological Principle and the debate about Large Scale Structures distribut

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1 Introduction

The basic hypothesis of a post-Copernican Cosmological theory is that *all the points* of the Universe have to be essentially equivalent: this hypothesis is required in order to avoid any privileged *observer*. This assumption has been implemented by Einstein in the so-called Cosmological Principles (CP): *all the positions* in the Universe have to be essentially equivalent, so that the Universe is homogeneous. This situation implies also the condition of spherical symmetry about every point, so that the Universe in also Isotropic. There is a hidden assumption in the formulation of the CP in regard to the hypothesis that all the points are equivalent on which we will return later.

The condition that all the occupied points are statistically equivalent with respect to their environment correspond to the property of of Local Isotropy. It is generally believed that the Universe cannot be isotropic about every point without being also homogeneous [1]. Actually Local Isotropy does not necessarily implies homogeneity; in fact a topology theorem states that homogeneity is implied by the condition of local isotropy together with *the assumption of the analyticity or regularity* for the distribution of matter.

Up to the seventies analyticity was an obvious implicit assumption in any physical problem. Recently however we have learned about intrinsically irregular structures for which analyticity should be considered as a property to be tested with appropriate analysis of experiment ([2],[3]).

The current idea is that in the observable Large Scale Structure distribution, isotropy and homogeneity do not apply to the Universe in detail but only to a "smeared- out " Universe averaged over regions of order λ_0. One of the main problems of observational cosmology is therefore the identification of λ_0.

Several recent galaxy redshift surveys such as CfA 1 [4], CfA 2 [5], [6], SSRS1 [7], SSRS2 [4] Perseus Pisces [8] and also pencil beams surveys [9],

have uncovered remarkable structures such as filaments, sheets, superclusters and voids. These galaxy surveys now probes scales up to $200-300h^{-1}Mpc$ and show that the Large Scale Structures are relatively common.

One of the most important issues raised by these catalogues is that the scale of *the largest inhomogeneities* is comparable with the extent of the surveys and that the largest known structures are limited by *the boundaries of the survey* in which they are detected. Da Costa et al. [6] emphasize that both CfA2 and SSRS2 contain voids with diameters as large as $50h^{-1}Mpc$ and that the Southern Survey contains the Southern Wall, similar to the Great Wall of the northern CfA: the general nature of the LSS distribution in the Southern Survey is similar to the Northern one. Moreover Da Costa et al.[6] examining the normalized density fluctuations in the CfA2 and SSRS2 surveys, concluded that there are fluctuations in shells of $100h^{-1}Mpc$ and that the combined sample is not homogenous.

Finally it is remarkable to note that Tully et al.[10] analyzing the combined Abell and ACO clusters catalogues provide that there are evidences of structures on scale of $450h^{-1}Mpc$ lying in the plane of the Local Supercluster.

This observational situation appears therefore highly problematic with respect to the identification of the length λ_0 above which the distribution should be smooth and essentially structureless.

2 Observations and analysis

Coleman and Pietronero [11], analyzing with the methods of modern statistical mechanics the Cfa 1 [4] for galaxies and the Abell catalogue [12] for clusters, find that these samples shows power law (fractal) correlations up to the sample limit without any tendency towards homogenization. In particular they find that the number-number correlation function (or other related quantities):

$$G(r) = < n(\vec{r_0}) n(\vec{r_0} + \vec{r}) > \tag{1}$$

has power law behaviour:

$$G(r) \sim A r^{-\gamma} \tag{2}$$

with $\gamma \sim 1.6-1.8$. If the distribution would really become homogenous at some length-scale λ_0 within the sample, one should instead observe a power

law followed, for $r > \lambda_0$, by *a well defined flat behaviour.*

The usual analysis consists in computing the two-point correlation function [13]:

$$\xi(r) = \frac{< n(\vec{r_0}) n(\vec{r_0} + \vec{r}) >}{< n >^2} - 1 \tag{3}$$

where $n(r)$ is the galaxy number density and $< n >$ is the average over the entire sample. At small scales one observes a power law behaviour [14]:

$$\xi(r) \sim A r^{-\gamma} \tag{4}$$

where $\gamma \sim 1.7 - 1.8$ both for galaxies and clusters catalogues. By the way we note that comparing eq.(1) and (3) it results that in the case of self-similar distribution, $\xi(r)$ is a power law only when $\xi(r) \gg 1$. The so-called "correlation length " in the standard approach is defined by the relation:

$$\xi(\vec{r_0}) = 1 \tag{5}$$

For galaxy distribution one obtains $r_0^G \sim 5Mpc$, while for clusters $r_0^C \sim 25Mpc$. From this analysis it follows that the amplitude of $\xi(r)$ is different for galaxies and clusters and the clusters are more correlated than galaxies. This observation has been one of the fundamental points of the so-called biased galaxy formation model [15].

The basic point of the debate about galaxies and clusters distributions and correlations [11] is that the analysis with the function $\xi(r)$ is meaningful only if the quantity $< n >$, that enters explicitly in the definition of $\xi(r)$ but not in $G(r)$, is a well defined quantity, i.e. it does not depend on the sample size. If, on the other hand, the distribution has correlations extending beyond the sample limit, the value of $< n >$ will be a direct function of the sample size. In this case the value of r_0 obtained via eq.(5) does not measure the real correlation length towards homogeneity, but it correspond just to a fraction of the sample size and it has no physical meaning with respect to the real correlation properties of galaxies and clusters. In other word the amplitudes of the correlation function of galaxies and clusters are not physical quantities but are related to the sample size.

Often the concept of *"fair sample"* is confused with "homogeneous sample" while these are two separate concepts. A sample is statistically fair if it is possible to extract from it information that is *statistically meaningful.*

Whether it is homogeneous or not is a property that can be tested and it is independent from statistical fairness.

For CfA 1 and Abell catalog Coleman and Pietronero [11] have obtained the following results:

- the CfA 1 catalogue is statistically a "fair sample" up to $20h^{-1}Mpc$:

- The correlation properties in the Cfa 1 show a well defined power law (fractal) behaviour up to the sample limits without any tendency towards homogenization.

- The Abell catalogue shows a well defined power law behaviour without any tendency towards homogeneity within the limits of statistical validity of the sample ($60-80h^{-1}Mpc$)

Now we can clarify that the mismatch between galaxy and clusters correlation is just due to an inappropriate mathematical analysis. The correlation lengths $r_0^G \sim 5Mpc$ for galaxies $r_0^C \sim 25Mpc$ for clusters are a finite portion of the depth of the galaxies (R_G) and clusters (R_C) catalogues. For Abell and Cfa 1 one finds that:

$$\frac{r_0^C}{r_0^G} \sim \frac{R_C}{R_G} \sim 5 \tag{6}$$

Therefore the correlation of clusters appear to be the continuation of galaxy correlations to larger scales *(Fig.1)* and the two samples simply refer to different observations of the system - which has fractal correlation up to the present observational limit. This amplitude rescale is in perfect agreement with the self-similar behaviour of galaxies and clusters distributions [11].

The fact that the amplitude of $\xi(r)$, and therefore also r_0, are larger for clusters than for galaxies has given rise to the statement that "clusters correlate to larger distance than galaxies". This proposition has been extended also among galaxies of different luminosity and the idea is that more luminous galaxies correlate to larger distances than less luminous ones. Since clusters are made of galaxies and voids are empty for both galaxies and clusters such statement is technically inconceivable. To clarify this point we recall that r_0 is a length related to the sample depth and has no physical meaning. Sample with more luminous galaxies are typically deeper than those with less luminous galaxies and this is the real origin of the effect. Therefore the *"galaxy-cluster mismatch"* and the *"clustering richness relation"* are problems that arise only from unreal quantities like the amplitude of $\xi(r)$ and r_0.

Figure 1: Ideal catalogue of galaxies and clusters. The correlation of clusters appear to be the continuation of galaxy correlation to larger scales: the points are galaxies and the groups of galaxies are identify as clusters. The depth of the clusters catalogues is deeper than that of galaxies catalogues because clusters are brighter than galaxies.

Within a correct analysis these problems automatically disappear and cluster correlations correspond naturally to galaxy correlations. The attempt of rescaling the two correlations via lower cut-off off length λ proposed by Szalay and Schramm [16], looses now its basic motivation: moreover this would not be the correct rescaling for self-similar system [11].

In addition to the homogeneity scale λ_0, it is interesting to identify the isotropy scale λ_I, i.e. the scale over which the statistical isotropy of the galaxy (or cluster) distribution has been reached. The evidence of dipole saturation in galaxies and clusters catalogues [17], together with a monotone growth of the monopole, shows that the isotropy scale has been reached, i.e. that λ_I is less than the catalogues depth. Since a fractal structure has usual the property of local isotropy, it is fully compatible with the evidence of dipole saturation with depth as well as an homogeneous distribution, as shown in

Figure 2: Monople and dipole modulus behaviour with sample depth for a fractal with $D = 1.4$: the evidence of dipole saturation is a proof of Local Isotropy and not of homogeneity

Fig.2 [18]. The isotropy scale cannot be simply related to the homogeneity scale unless the distribution is smooth (analytic), but we have seen that it is certainly not the case.

Very recently we have also analyzed the ESO Slice Project (ESP) [19] that is a galaxy redshift survey over a strip of $22° * 1°$ in the South Galactic Pole region and with a limiting magnitude of $b_J \leq 19.4$. The total number of objects is of the order of 4000. Due to the small solid angle covered by this survey it is not possible to compute the number-number correlation function, while it is possible to study the *number-redshift* relation $N(z)$, that gives the total number of galaxy within a spherical symmetric volume of redshift z. This is one of the crucial test for world models [20] and gives unambiguously, and free of any a priori assumption, the properties of the galaxy number density.Our preliminary analysis shows an indication that in the ESP survey the distribution of galaxies is not homogenous up to the sample depth ($\sim 500 - 700 h^{-1} Mpc$), and that it shows power law (fractal)

correlations up to the sample limits; the exponent of the fractal distribution is about 2.

The pictures that emerges with an analysis of galaxy and cluster redshift surveys, that does not imply any a priori assumption is quite different from the standard one: the larger scale distribution of galaxies and clusters shows well defined fractal properties without any tendency towards homogeneity *up to the present observational limits*.

3 Properties of a fractal distribution of matter

A non-analytical distribution can be statistically isotropic above some scale λ_I. This means that all the points are statistically equivalent with respect to their environment. A non-analytical (fractal) distribution breaks the symmetry between occupied and empty points: the translation invariance is not satisfied any more even if the distribution shows on average spherical symmetry around every point. This means that each point of the fractal structure is statistically identical (in the sense of local isotropy) to any other point of the structure. However in the middle of a void the environmental properties are different than for the points of the structure.

The cosmological principle can therefore be maintained in term of Local Isotropy in perfect agreement with the basic hypothesis of having no privileged points [2], see also [21].

The absence of analyticity has important consequences. First of all irregularity is intrinsic at all scales and the structure never becomes smooth (analytical). For this reason the global averages like number density or, as we have shown in the previous section, the amplitude of $\xi(r)$ are related to the sampled volume. In particular we stress that the normalized density fluctuations $\delta N/N$ at scale of $8h^{-1}Mpc$ suffers of the same problem of r_0 and its value is the counterpart of the of the finite size of the sample and has no direct physical meaning. Hence one cannot identify the scale at which the fluctuations are small respect to the average as the scale of the transition from non linear to non-linear behaviour: this scale is a consequence of an incorrect mathematical analysis and a fractal structure is "non-linear" at all

scales. However considering a finite portion of a fractal structure, one can always find a scale at which $\delta N/N \ll 1$, but this is just due the finite size of the system rather than to an intrinsic physical properties. The distance at which $\delta N/N = 1$ will scale with the sample depth, as r_0 scales [22], so it is not an evidence of homogeneity, or any other ral change of nature of the distribution.

4 The multifractal mass distribution

Coleman & Pietronero [11] have performed a multifractal analysis of the Cfa1 for the full matter distribution, by including also the galaxy masses: the result shows indeed well defined multifractal properties. This fact has a number of consequences on various morphological properties. Here we discuss the main one, i.e the correlation between space and mass distributions.

We have seen that a basic characteristic of the observable distribution of galaxies is the existence of LSS having fractal properties at least up to some length that increases with each new catalogue. A second important observational feature is the galaxy mass function: this function determines the probability of having a mass in the range M to $M + dM$ for unit volume, and can be described by the Press-Schecheter function that has a power law behaviour in the limit of low mass, followed by an exponential cut-off [23]:

$$n(M)dM \sim M^{\delta-2} exp(-(M/M^*)^{2\delta})dM \tag{7}$$

with $\delta \sim 0.2$. These two observational evidences can be linked together by the concept of multifractal (MF) that gives an unified picture of the mass and spatial distributions. A MF distribution describes systems with local properties of self-similarity [24] and it is characterised by a continuous set of exponent $f(\alpha)$. This distribution implies a strong correlation between spatial and mass distributions so that the number of object with mass M for unit volume $\nu(M, \vec{r})$, is a complex function of space and mass and is not simply separable in a density function $D(\vec{r})$ multiplied by a mass function $n(M)$, as usually done.

It can be shown [25] that the mass function of a multifractal in a well defined volume, has a Press-Schechter shape with the exponent δ that depends on the shape of $f(\alpha)$. Moreover the fractal dimension of the support

is $D(0) = f(\alpha_s) = 3 - \gamma$ (eq.(2)). Hence with the knowledge of the whole $f(\alpha)$ spectrum one obtains information on the space correlation and on the mass function.

The evidence that galaxian luminosity and location in space are not independent and that, on the contrary, are strongly correlated have important consequences in all the luminosity properties of visible matter and in relation to the methods that are usually used to study these properties [11], [25].

From a theoretical point of view the problem is therefore to identify the dynamical processes that leads to such a multifractal distribution, as discussed in [26] on this issue.

5 Conclusion

The new experimental picture that comes out from the redshift survey is that the distribution of visible matter is fractal and multifractal up to the present observational limits (see *Fig.3*). New deep redshift surveys as the ESP survey [20], but also pencil beam surveys [9], suggest that the cut-off towards homogeneity is deeper than the survey limit $\sim 500 - 600 h^{-1} Mpc$.

We have shown that a fractal (and multifractal) distribution is, of course, not homogenous and can be locally isotropic so that it holds the basic hypothesis that all the points of the Universe are essentially equivalent and that there are not privileged observer. The Cosmological Principle can be naturally generalized to the case of non-analytical distribution that breaks the symmetry between occupied and empty points.

The relation of the new picture to the metric and Einstein's equation depends crucially on the eventual properties of the dark matter. If this would turn out to be usual and distributed homogeneously there is basically no problem with the predominant description of the metric, the Big-Bang model, etc. In the opposite situation the problem becomes very hard and it has to be reconsidered from the beginning. In any case some fundamental aspects of the currently theories of galaxy formation, such as the biased galaxy formation model and related theories, have fundamental problems with the new picture that we have briefly described.

Figure 3: The large scale distribution of visible matter in the universe according to the usual picture (above) compared to the one that aries from our new studies (below).

The usual discussion of biased galaxy formation is implemented by arguing that before and after the evolution of fluctuations one has two different types of density fluctuations both however within an analytic Gaussian framework. Thus, neither long range correlation are present nor the power law behaviour. The self-similarity and the non-analyticity correspond to a breakdown of the Central Limit Theorem which is instead the necessary cornerstone of gaussian process. In addition a power law's amplitude has virtually no meaning while the exponent is the crucial quantity, so that one has to approach the problem with a fundamentally different theoretical framework (see for example [3])

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